CHURCHLANDS
SENIOR HIGH SCHOOL

MATHEMATICS SPECIALIST 3,4
TEST 2 SECTION TWO 2016
Calculator Section
Chapters 3 and 4

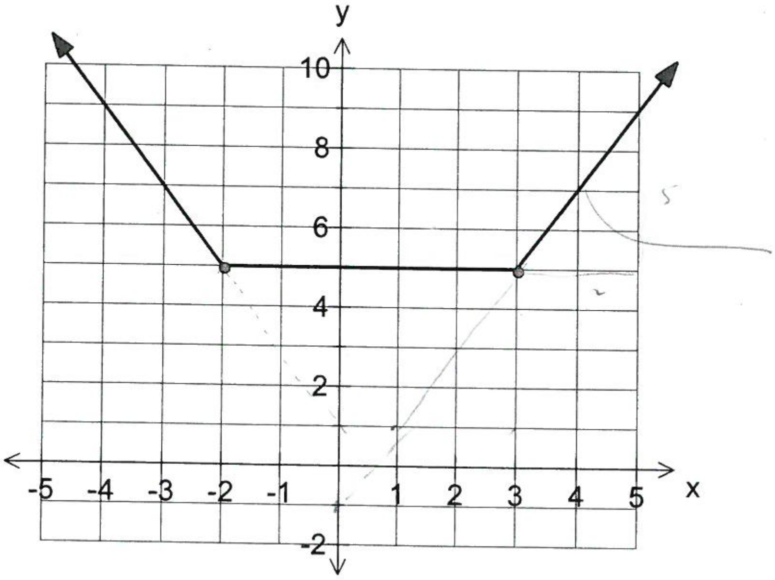
Name________________________________

Time: 20 minutes
Total: 20 marks

Question 1

(5 marks)

The function *f*, defined for all real *x* by $f(x) = |x-a| + |x+b|$, where *a* and *b* are positive integers, has the following graph.



a = 3

b = 2

$2x + c = y$

$6 + c = 5$

$c = -1$

(a) Find the values of *a* and *b*.

a = 3 b = 2 ✓✓

$f(x) = |x-3| + |x+2|$

(b) Express $f(x)$ as a piecewise function.

$$f(x) = \begin{cases} -2x+1, & x < -2 \ ✓ \\ 5 & -2 \le x \le 3 \ ✓ \\ 2x-1 & x > 3 \ ✓ \end{cases}$$

Question 2 **(5 marks)**

At 10.00am, two bumper cars at the royal show, G and T, have position vectors, ***r*** m, and velocity vectors, ***v*** m/s, as shown below:

$\mathbf{r}_G = 3\mathbf{i} + 9\mathbf{j}$ $\mathbf{v}_G = -\mathbf{i} - \mathbf{j}$

$\mathbf{r}_T = 9\mathbf{i}$ $\mathbf{v}_T = -5\mathbf{i} + 5\mathbf{j}$

Prove that the bumper cars will collide if they continue with these velocities and find the time and location of the collision.

$\underset{\sim}{r}_G = 3\underset{\sim}{i} + 9\underset{\sim}{j} + t(-\underset{\sim}{i} - \underset{\sim}{j})$ $r_T = (9 - 5t)\underset{\sim}{i} + (5t)\underset{\sim}{j}$

$= (3 - t)\underset{\sim}{i} + (9 - t)\underset{\sim}{j}$

For coll

$\therefore$ $3 - t = 9 - 5t$ $9 - t = 5t$

$4t = 6$ ✓ $9 = 6t$

$t = 1.5$ $1.5 = t$ ✓

$\therefore$ collision occurs at 1.5 seconds ✓

Time 10.00 + 1.5 seconds ✓

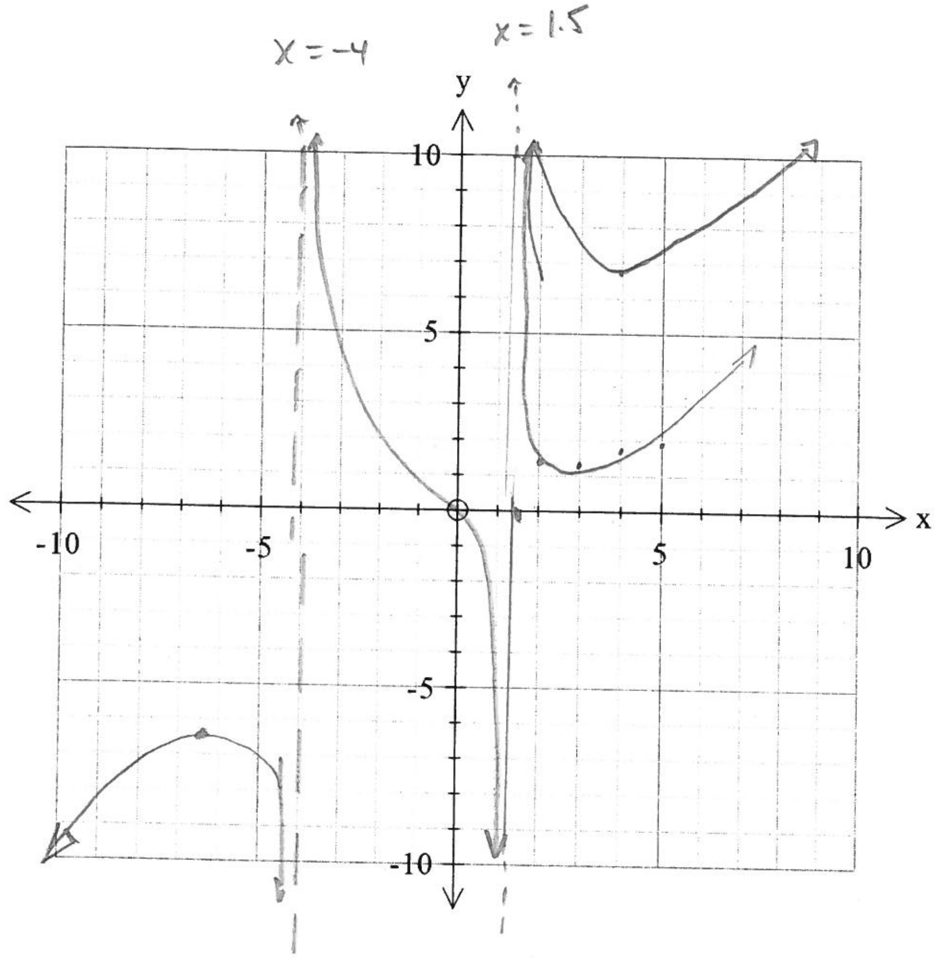
location: $= 9 - 5(1.5)\underset{\sim}{i} + 5 \times 1.5\,\underset{\sim}{j}$

$= 1.5\underset{\sim}{i} + 7.5\underset{\sim}{j}$ ✓

$\neq$ [illegible]

Question 3 **(5 marks)**

Sketch the graph $y = \dfrac{x^3}{(x+4)(2x-3)}$., the asymptotes and describe the behaviour of the graph as $x \to \pm\infty$. Give the equations for the vertical and other asymptotes.



asympt ✓

correct shape ✓

correct TPs ✓

$x = -4, \ x = 1.5$ ✓

$y = \frac{1}{2}x$ ✓

$$y = \frac{x^3}{2x^2 + 5x - 12}$$

as $x \to \pm\infty$ can consider dominant terms.

ie $y \simeq \dfrac{\pm x^3}{2x^2}$

$\simeq \pm \dfrac{x}{2}$

$= \pm \dfrac{1}{2}x$

Oblique asymptote of $\underline{y = \frac{1}{2}x}$

Question 4 **(5 marks)**

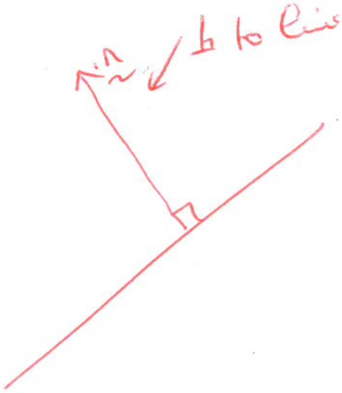
Find the Cartesian equation of the line perpendicular to the vector $7\underset{\sim}{i}+5\underset{\sim}{j}$ and passing through the point $(-1,3)$

$$\underset{\sim}{r}\cdot\underset{\sim}{n} = \underset{\sim}{a}\cdot\underset{\sim}{n} \quad ✓$$

$$\underset{\sim}{r}\cdot\left(7\underset{\sim}{i}+5\underset{\sim}{j}\right) = \left(-1\underset{\sim}{i}+3\underset{\sim}{j}\right)\cdot\left(7\underset{\sim}{i}+5\underset{\sim}{j}\right) \quad ✓$$

$$\underset{\sim}{r}\cdot\begin{pmatrix}7\\5\end{pmatrix} = 8 \quad ✓$$

now $\underset{\sim}{r} = \begin{pmatrix}x\\y\end{pmatrix}$



$$\therefore \quad 7x+5y = 8 \quad ✓$$

$$5y = -7x+8$$

$$y = -\frac{7}{5}x+\frac{8}{5} \quad ✓$$

<u>OR</u> // to line is $-5\underset{\sim}{i}+7\underset{\sim}{j}$

$$\therefore \underset{\sim}{r} = \langle -1, 3\rangle + \lambda\langle -5, 7\rangle$$

$$= \langle -1, -5\lambda\rangle + \langle 3+7\lambda\rangle$$

$$\therefore \quad x = -1-5\lambda \qquad y = 3+7\lambda$$

$$\lambda = \frac{x+1}{-5} \qquad \lambda = \frac{y-3}{7}$$

$$\Rightarrow \quad 7x+7 = -5y+15$$

$$\underline{7x+5y = 8}$$

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MATHEMATICS SPECIALIST 3,4
TEST 2 SECTION ONE 2016
NON Calculator Section
Chapters 3 and 4

Name________________________________

Time: 35 minutes
Total: 35 marks

Question 1 **(7 marks)**

Two functions are defined as $f(x)=\sqrt{x-1}$ and $g(x)=\dfrac{1}{x-1}$

(a) Evaluate $gf\left(\dfrac{13}{9}\right)$ (2 marks)

$$= g\left(\sqrt{\tfrac{13}{9}-1}\right)$$

$$= g\left(\sqrt{\tfrac{4}{9}}\right)$$

$$= g\left(\tfrac{2}{3}\right) \quad ✓$$

$$= \frac{1}{\tfrac{2}{3}-1} = -3 \quad ✓$$

(b) Find in simplified form $gg(x)$. (2 marks)

$$g\left(\frac{1}{x-1}\right) = g\left(\frac{1}{\frac{1}{x-1}-1}\right) \quad ✓$$

$$= \frac{1}{\frac{1-(x-1)}{x-1}} = \frac{x-1}{2-x} \quad ✓ \left\{\frac{x}{1-x}\right\}$$

(c) Determine the domain of $f(g(x))$ (3 marks)

$$= \sqrt{\frac{1}{x-1}-1}$$

need $\dfrac{1}{x-1}-1 \geq 0$

$$\frac{1-(x-1)}{x-1} \geq 0$$

$$\frac{2-x}{x-1} \geq 0 \quad ✓$$

$\therefore$ Domain $1 < x \leq 2$

✓ ✓

Question 2 **(6 marks)**

(a) Determine the domain and range of $f(g(x))$ given that $f(x) = \dfrac{12}{x+1}$ and $g(x) = \sqrt{x+1}$ (3)

$$f\,g(x) = \frac{12}{\sqrt{x+1}\ \ +1} \quad ✓$$

$$D : \{x : x \in R \,.\, x \geq -1\} \quad ✓$$

$$R : \{y : y \in R \; 12 \geq y \geq 0\} \quad ✓$$

(b) Given that $f(x) = 2x+3$ and $g(f(x)) = 4x^2 + 12x + 11$, find $g(x)$. (3)

Let $K = 2x+3 \Rightarrow x = \dfrac{K-3}{2}$

$$g(K) = 4\left(\frac{K-3}{2}\right)^2 + 12\left(\frac{K-3}{2}\right) + 11 \quad ✓$$

$$= 4\left(\frac{K^2 - 6K + 9}{4}\right) + 6(K-3) + 11 \quad ✓$$

$$= K^2 - 6K + 9 + 6K - 18 + 11$$

$$= K^2 + 2$$

$$\therefore g(K) = \underline{K^2 + 2} \quad ✓$$

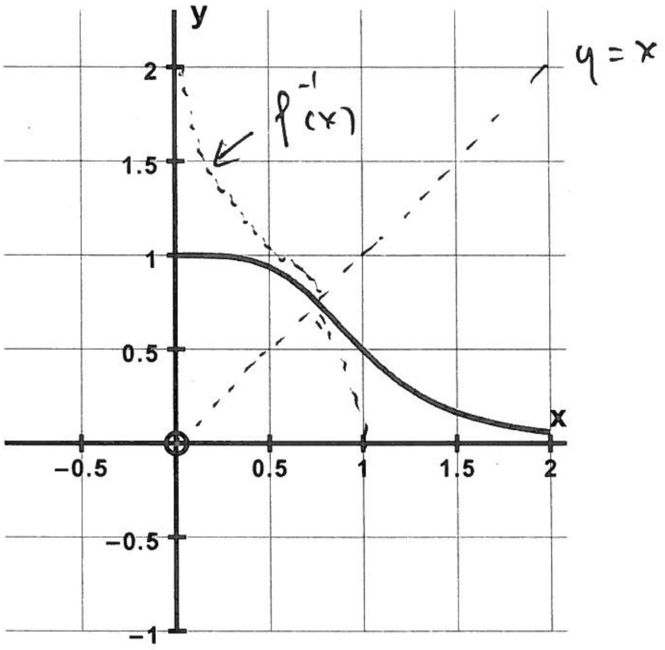
$$\begin{array}{r} 2x+3 \\ 2x+3 \overline{) \; 4x^2 + 12x + 11} \end{array}$$

$\underline{\underline{r = 2}}$

ie square it then add $\underline{2}$

Question 3 **(6 marks)**

The graph of function $f(x) = \dfrac{1}{x^4+1}$ for the domain $0 < x < 2$ is shown below.



(a) Determine the exact value for $\lim\limits_{x \to 2^+} f(x)$ $= \dfrac{1}{2^4+1}$ ✓ $= \dfrac{1}{17}$ ✓ (2)

(b) On the axes given above, sketch the graph of the inverse function, $y = f^{-1}(x)$ (2)

✓1 Reflect in $y = x$

✓1 pt of intersect at $\underline{\underline{x = y}}$

(c) Obtain the rule for $f^{-1}(x)$. (2)

$f^{-1}(x) = \sqrt[4]{\dfrac{1}{x} - 1}$ or

$y = \dfrac{1}{x^4+1}$

$x = \dfrac{1}{y^4+1}$

$y^4 + 1 = \dfrac{1}{x}$

$y^4 = \dfrac{1}{x} - 1$

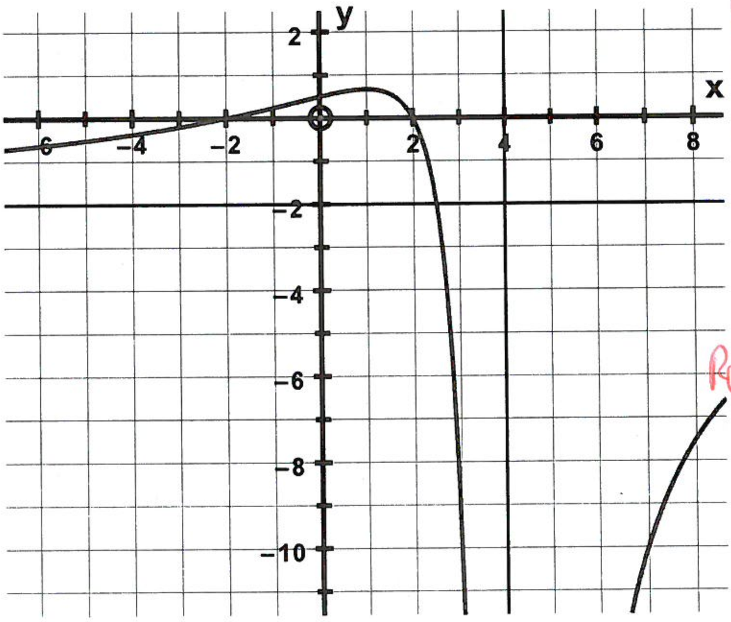
$y = \sqrt[4]{\dfrac{1}{x} - 1}$

$\left\{ = \sqrt[4]{\dfrac{1-x}{x}} \right\}$

Question 4 **(5 marks)**

A rational function $R(x)$ is sketched below. Function $R(x)$ has the following properties:

- Only one pole or a discontinuity at $x = 4$
- Two horizontal intercepts at $x = 2$ *and* $x = -2$.
- A horizontal asymptote at $y = -2$



For a

$k(x^2 - a) = 0$ for $R(x) = 0$

$\Rightarrow x^2 - a = 0$

this occurs when

$x = +2$ or -2 intercepts

$2^2 - a = 0 \Rightarrow a = \underline{4}$ ✓✓

$$R(x) = \frac{k(x^2 - 4)}{(x-b)(x-c)} = \frac{k(x-2)(x+2)}{(x-b)(x-c)}$$

Discont at $x = 4$ <u>only</u> ✓

$\Rightarrow$ Denominator

$(x-b)(x-c)$ $\underline{b = c = 4}$

(a) If $R(x) = \dfrac{k(x^2 - a)}{(x-b)(x-c)}$ explain why $k = -2,\ a = 4, b = 4$ *and* $c = 4$

$$\lim_{x \to \infty} R(x) = \underline{\underline{-2}}$$

✓ considering dominant terms

$$\therefore \frac{kx^2 - \cancel{ka}}{x^2 - \cancel{xb} - \cancel{xc} + \cancel{bc}} \approx \frac{k\cancel{x^2}}{\cancel{x^2}}$$

$$\approx k \underline{= -2}$$

$x = \pm 2 \rightarrow$ intercepts,

so $x^2 - a = \cancel{(x+n)(x-n)}\ 0$

diff of squares

so $a = 4$ $\Rightarrow a \pm 4$

(4)

or. when $x = 3$ $y = -10$

$\therefore \underset{+1}{\underline{5}}\, k = -10 \Rightarrow k = -2$

(b) Determine $\lim\limits_{x \to 4} R(x)$. Does not exist

(1)

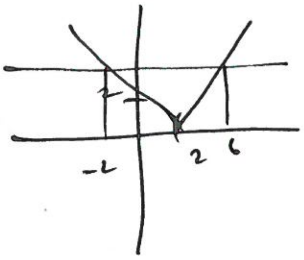
Question 5 **(7 marks)**

Solve the following.

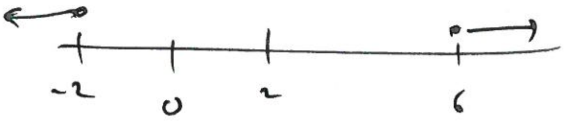
(a) $|x-2|>4$ (1)

$x-2=4 \Rightarrow \underline{x=6}$

$-x+2=4 \Rightarrow \underline{x=-2}$ ✓ ∴



or dist fr. 2 to any pt is greater than 4



or

$(x-2)^2 = 4^2$

$x^2-4x+4=16$

$x^2-4x-12=0$

$(x\ [illegible]\ 6)(x+2)=0$

$\therefore x = +6$

$\underline{x=-2}$ ✓

$\therefore x>6,\ x<-2$

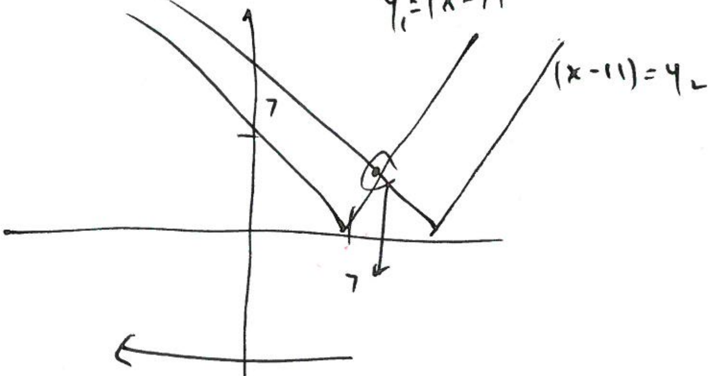
(b) $|x-7| \le |x-11|$ (2)

$(x-7)^2 \le (x-11)^2$

$x^2-14x+49 \le x^2-22x+121$

$8x \le 72$ ∴

$\underline{x \le 9}$



graph of $y_2 \geq y_1$

solve $-(x-11) \geq x-7$

$-x+11 \geq x-7$

$18 \geq 2x$

$\underline{9 \geq x}$

(c) $|3x+4| \geq |5x+2|$ (2)

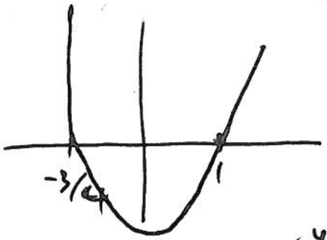
$(3x+4)^2 \geqslant (5x+2)^2$

$9x^2 + 24x + 16 \geqslant 25x^2 + 20x + 4$

$0 \geqslant 16x^2 - 4x - 12$

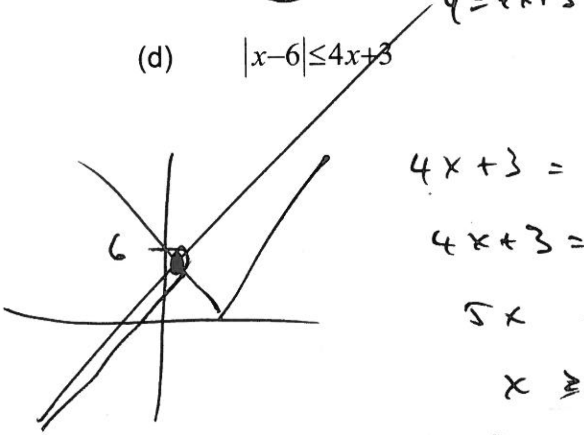
$0 \geqslant 4x^2 - x - 3$

$0 \geqslant (4x+3)(x-1)$



$-\frac{3}{4} \leq x \leq 1$

(d) $|x-6| \leq 4x+3$ (2)



$4x+3 = -(x-6)$

$4x+3 = -x+6$

$5x = 3$

$x \geq 3/5$ /

Issues!!

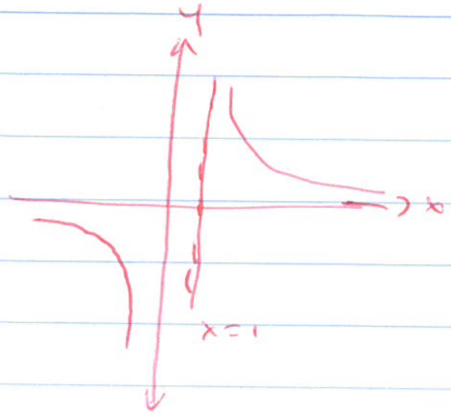
$x^2 - 12x + 36 \leq 16x^2 + 24x + 9$

$0 \leq 15x^2 + 36x - 27$

$0 \leq 5x^2 + 12x - 9$

$(5x-3)(x+3)$

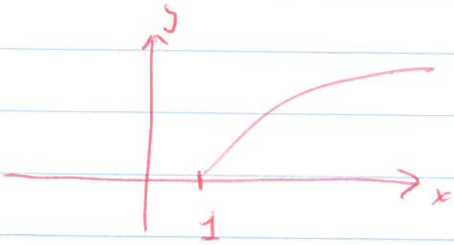
$fg(x)$



now $g(x) = \frac{1}{x-1}$

$D_g : \{x \in R : x \neq 1\}$

$R_g : \{y \in R : y \neq 1\}$



$f(x) = \sqrt{x-1}$

$D_g : \{x \in R : x \geq 1\}$

now the range of $g(x)$ will be the domain of f in $fg(x)$

so f cannot has a natural Domain $x \geq 1$

But $g(x)$ cannot output 1 as $y \neq 1$

so x must be either > 1 or < 1

Now $fg(x) = \sqrt{\frac{1}{x-1} - 1}$

Note $\frac{1}{x-1} - 1 \geq 0$

$\frac{1}{x-1} \geq 1$

$\cancel{x \geq} 1 \geq x - 1 \Rightarrow 2 \geq x$